

# Electric Dipole Moment of Dirac Fermionic Dark Matter

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## Abstract

The direct limit of electric dipole moment (EDM) and direct search for dark matter by EDM interaction are considered as including the electromagnetic nuclear form factor, in case that the dark matter candidate is a Dirac particle. The WIMP electric dipole moment constrained by direct searches must be lower than  $7 \times 10^{-22} e \text{ cm}$  for WIMP mass of 100 GeV to satisfy the current experimental exclusion limits at XENON10 and CDMS II. We also consider the CP violation of EDM and the WIMP discovery by EDM interaction in the future.

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## I. INTRODUCTION

Dark matter (DM) has been postulated to explain various observations from gravitational effects on visible matter and plays an important role to explain structure formation and galaxy evolution. In recent years observations and the high precision analysis of the cosmic microwave background radiation have provided spectacular confirmation of the astrophysical evidence for DM, that is to say about quarter of the energy density of the universe is dark matter. Dark matter appears to be consisting of nonrelativistic particles that only interact gravitationally and perhaps by weak interaction. Mostly the coupling to photons is assumed to be nonexistent or very weak, so the electromagnetic interactions have not been considered seriously. A possible scenario for electromagnetic interaction of Dirac fermionic dark matter with nonzero magnetic dipole moment [1] has been considered in the standard model context. No additional particles are assumed except for the DM candidate near electroweak scale ( $10 \sim 1000$  GeV). The various experimental bounds was investigated [2, 3, 4] in the past and a new experimental technique [5] for DM detection by electromagnetic interaction has been suggested for nonzero dipoles. There is a DM scenario for the electromagnetic interaction by the fractional or millie charged particles [7, 8], but the DM scenario is for the case that another massless  $U(1)$  gauge boson, called paraphoton, exists beyond the standard model.

In this letter we consider electric dipole moment effect of Dirac fermionic dark matter. The electric dipole moment (EDM) constrained by direct searches is investigated as considering the electromagnetic form factor of the nucleus. The WIMP-nucleus elastic scattering is due to spin independent interaction, that gives the WIMP electric dipole moment very strict bound since the WIMP-nucleus elastic scattering cross sections are enhanced by the square of nuclear charge (number of protons in the nucleus),  $Z^2$ . The WIMP electric dipole moment constrained by direct searches must be lower than  $7 \times 10^{-22} e \text{ cm}$  for WIMP mass of 100 GeV to satisfy the current experimental exclusion limits. WIMP electric dipole moment is scaled as considering the current experimental exclusion limit of electric dipole moment (EDM) [6] for the known Dirac particles and the scenario that presented in Ref.[1] to investigate the WIMP detectability. A simple model (Lagrangian) with the complex dipole coupling is also introduced. Although we consider that the interaction of EDM is suppressed by CP violation, the suppression could be compensated by the enhancement of spin independent interaction. WIMP could thus be detected by the EDM interaction in near future if the

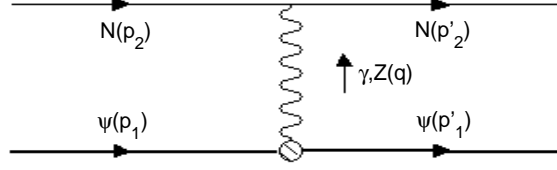


FIG. 1: Feynman diagram relevant to WIMP-nucleon elastic scattering. The hatched circle indicates the vertex for the dipole coupling.

suppression is not seriously small.

## II. CONSTRAINTS FROM DIRECT SEARCHS

The detection of dark matter is controlled by their elastic scattering with a nucleus in a detector. In this case the  $t$ -channel exchange of a photon<sup>1</sup> is only possible as shown in Fig. 1, since the two interacting particles are distinguishable.

The effective Lagrangian at the quark level may be described by

$$\mathcal{L}_q = \frac{de e_q}{q^2} (\bar{\psi} \sigma^{\mu\nu} q_\nu \gamma^5 \psi) (\bar{q} \gamma_\mu q), \quad (1)$$

where  $d$  is the WIMP electric dipole moment,  $e$  is the electric coupling and  $e_q$  is an electric charge for the quark  $q$ .

In nonrelativistic case, the bi-spinor products may be expanded with respect to the low momentum transfer. In the leading order of the momentum transfer, only time component is taken.

The effective interaction of WIMP with a nucleon is

$$\mathcal{L}_N \supset 2 \frac{de f_N}{q^2} M (\bar{\psi} \gamma^5 \psi) (N^\dagger N), \quad (2)$$

where  $N$  stands for the proton (p) or neutron (n) and  $f_N$  is the effective coupling of WIMP to nucleons. In this case,  $f_p = 2e_u + e_d = 1$ ,  $f_n = e_u + 2e_d = 0$ . The contributions of the heavy quark ( $c, b$  and  $t$ ) to WIMP-nucleon cross section can be related to the gluon

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<sup>1</sup> A  $Z$ -boson exchange is also possible in the DM scenario of Ref.[1], but the interaction by a  $Z$ -boson exchange is negligible for the low momentum transfer.

contribution by QCD, but those appear in quark-antiquark pairs. So three valance quarks give the nucleon its global electric charge and the contributions of each valance quark in the nucleon add coherently.

The similar argument can be applied to the nuclei. The constructive coherent interactions give the squared scattering amplitude of the nucleus

$$|\mathcal{M}|^2 = 16 \frac{d^2(Ze)^2 M^2 m^2}{\mathbf{q}^4} \frac{1}{2S+1} \sum_{\text{spins}} (\mathbf{q} \cdot \mathbf{S})^2 F^2(|\mathbf{q}|), \quad (3)$$

where  $S = \frac{1}{2}$  is the WIMP spin,  $Z$  is the nuclear electric charge (the number of protons in the nucleus) and  $F^2(|\mathbf{q}|)$  is the electromagnetic form factor that is related to the electric charge distribution in a nucleus. For the low momentum transfer, we cannot consider the large nuclei as a point particle. We need consider the charge distribution of the nucleons in a nucleus. We take the Helm form factor [10] that was introduced as a modification of the form factor for an uniform sphere multiplied by a gaussian to account for the soft edge of the nucleus.

$$F^2(|\mathbf{q}|) = \left[ \frac{3j_1(|\mathbf{q}|R_1)}{|\mathbf{q}|R_1} \right]^2 \exp[-(|\mathbf{q}|s)^2], \quad (4)$$

where

$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$$

is a spherical Bessel function of the first kind, and where  $s \simeq 1$  fm is the nuclear skin thickness and  $R_1 = \sqrt{R^2 - 5s^2}$  is an effective nuclear radius for nuclear radius  $R \simeq 1.2$  fm  $A^{1/3}$  for an atomic nucleus of atomic number  $A$ . This form factor is often referred as the "Woods-Saxon" form factor though this form factor is not that obtained from the Fourier transform of the Woods-Saxon density distribution.

The WIMP-nucleus differential cross section results in

$$\frac{d\sigma_{el}}{d\mathbf{q}^2} = \frac{2\alpha Z^2 d^2}{v^2 \mathbf{q}^2} F^2(|\mathbf{q}|), \quad (5)$$

where  $\alpha = e^2/4\pi \simeq 1/137$  is the electric fine structure constant. The cross section has the Coulomb-like singularity at  $\mathbf{q}^2 = 0$ .

The expected event rate depends on WIMP-nucleus cross section and the WIMP flux on the Earth, so the event rate per unit target mass and unit time is

TABLE I: Current and planned Dark Matter detectors. XENON10 and CDMS II are the current detectors and SuperCDMS is a planned detector.

Experiment	Recoil energy range	Target	Nuclear charge (Z)	Mass
XENON10	4.5 $\sim$ 27KeV	$^{131}\text{Xe}$	54	5.4Kg
CDMS II	10 $\sim$ 100KeV	$^{73}\text{Ge}$	32	100Kg
SuperCDMS	15 $\sim$ 45KeV	$^{73}\text{Ge}$	32	100 Kg

$$\frac{dR}{dE_R} = \frac{\rho_D}{mM} \int v f(v) \frac{d\sigma_{el}}{dE_R} dv, \quad (6)$$

where  $\rho_D \simeq 0.3 \text{ GeV/cm}^3$  is the local DM density in the solar vicinity and  $f(v)$  is the WIMP velocity distribution funtion in the frame of the detector.

With the relation  $\mathbf{q}^2 = 2mE_R$ , the differential event rate is

$$\frac{dR}{dE_R} = \frac{\alpha Z^2 d^2 \rho_D}{mM} \frac{F^2(E_R)}{E_R} \int \frac{f(v)}{v} dv. \quad (7)$$

The final formula for the event rate per unit mass and unit time is given by

$$R = \frac{\alpha Z^2 d^2 \rho_D}{mM} \int_{E_{R,\min}}^{E_{R,\max}} dE_R \frac{F^2(E_R)}{E_R} \cdot \frac{1}{2v_E} \left[ \text{erf} \left( \frac{v_{\min} + v_E}{v_0} \right) - \text{erf} \left( \frac{v_{\min} - v_E}{v_0} \right) \right], \quad (8)$$

with  $v_{\min} = \sqrt{\frac{E_R m}{2M_\tau^2}}$ ,  $v_0 = 220 \text{ km/s}$  is the circular speed of the Sun around the Galactic center and  $v_E = 232 \text{ km/s}$  is the average velocity considered the Earth speed to the Sun. Considering the recoil energy ranges at Table I, the electric dipole moment of a WIMP constrained by direct searches must be lower than  $7 \times 10^{-22} e \text{ cm}$  for WIMP mass of 100 GeV.

### III. ELECTRIC DIPOLE EFFECT

#### A. Theory of Dipole Moments

The annihilation rates by EDM interaction are suppressed since the s-wave amplitude is zero due to CP violation, so EDM interaction cannot produce the right magnitude of the relic density. To reach the thermal average of annihilation rate  $\langle \sigma v_{\text{rel}} \rangle \simeq 0.62 \text{ pb}$ , EDM has to be

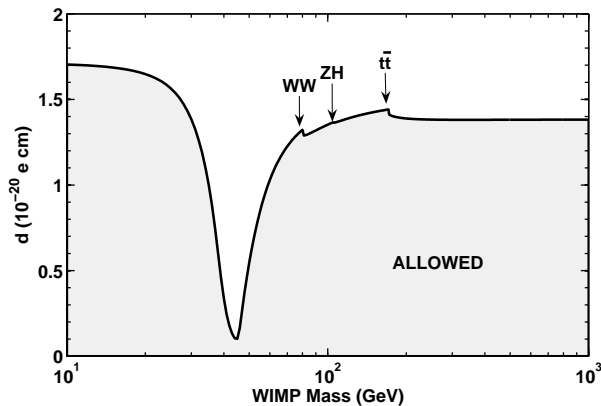


FIG. 2: Allowed parameter regions for  $\epsilon = d/\mu < 2 \times 10^{-3}$ . The arrows indicate the points where new channels start.

very large ( $\sim 10^{-16} e cm$ ) and it is ruled out by direct searches for WIMP near electroweak scale. Simply, we give the relation between EDM and MDM,  $d = \epsilon\mu$ , where  $\mu$  is MDM, and we consider the minimal Dirac fermionic dark matter scenario with nonzero MDM [1]. With this relation, the parameter  $\epsilon$  becomes a free parameter that would be constrained by EDM CP related phenomenology. The most simple model (Lagrangian) is with the complex dipole coupling. If the loop contributions to all the electromagnetic dipole operators have very similar diagrammatic structure <sup>2</sup>, the effective Lagrangian may be represented with the complex dipole coupling  $\mathcal{D}$ .

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\mathcal{D}\bar{\psi}\sigma_{\mu\nu}P_L\psi F^{\mu\nu} + \frac{1}{2}\mathcal{D}^*\bar{\psi}\sigma_{\mu\nu}P_R\psi F^{\mu\nu} = \frac{1}{2}\bar{\psi}(\mu - id\gamma_5)\sigma_{\mu\nu}\psi F^{\mu\nu}, \quad (9)$$

where  $P_L, P_R$  are the left and right handed projectors and  $F^{\mu\nu}$  is the electromagnetic field strength. The real part of the complex dipole coupling is consistent with magnetic dipole and the imaginary part is for electric dipole. The relations with real dipoles are  $\mu = |\mathcal{D}|\cos\phi$  and  $d = |\mathcal{D}|\sin\phi$ , where  $\phi$  is the phase of electromagnetic dipole operators. The relationship between dipoles is given by  $d = \mu \tan\phi$  and  $\epsilon = \tan\phi$  would account for all the phenomenology for CP violation of EDM.

The phase of dipole operators would be constrained by the exclusion limit of EDM [6] and anomalous MDM for the known Dirac particles like muon or electron,  $\epsilon = \tan\phi \leq 2 \times 10^{-3}$ .

<sup>2</sup> Actually this was pointed out in Ref. [9] for the ordinary Dirac particles in supersymmetric theories.

Our candidate might not be amenable to this constraint, but we adopt this constraint to analyze EDM effect for direct searches. Fig.2 shows the allowed parameter regions of EDM in mass range  $10 \sim 1000$  GeV. The bold line on the border is for  $\epsilon = 2 \times 10^{-3}$ , and MDM is taken in Ref.[1], that is constrained by the relic density.

## B. WIMP Detectability

The cross sections are enhanced by the square of the nuclear charge  $Z^2$  and this effect might be able to compensate the suppression of EDM CP violation. Fig.3 shows the regions of the the expected event rates of Eq.(8) with the estimated EDM from the above argument. The experimental exclusion limits of XENON10, CDMS II and SuperCDMS are included. The experimental sensitivities estimated by the current detectors, XENON10 [12] and CDMS II [13], are for 0.1 cpd/kg and the  $1 \times 10^{-4}$  cpd/kg exclusion limit is considered for the planned detector, SuperCDMS [14], respectively. Traditionally, the results of the direct searches are presented in the form of the WIMP-nucleon cross section at the zero momentum transfer in the spin independent case, since such normalized forms are useful to compare results for different types of nuclei. But the cross section is not defined at the zero momentum transfer in this case, so we consider the predicted event rates and the experimental exclusion limits for event rates that estimated for the detectors.

The predictions are over exclusion limits for  $\epsilon = d/\mu = 2 \times 10^{-3}$  and WIMP could be detected in near future by EDM interaction if CP violation of EDM is not seriously small. The discovery of WIMP by EDM interaction can also be the key to disclose CP violation nature of EDM.

## IV. CONCLUSION

The direct limit of electric dipole moment (EDM) and direct search for dark matter by EDM interaction has been considered in case that the dark matter candidate is a Dirac particle. The WIMP-nucleus elastic scattering is due to spin independent interaction, that gives the WIMP electric dipole moment very strict bound since the WIMP-nucleus elastic scattering cross sections are enhanced by the square of nuclear charge (number of protons in the nucleus),  $Z^2$ . The WIMP electric dipole moment constrained by direct searches must

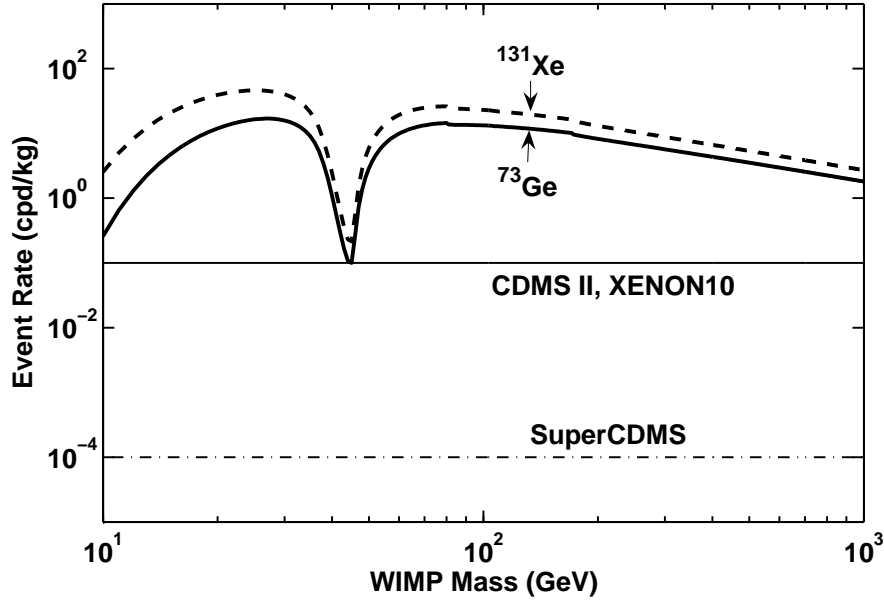


FIG. 3: The expected event rates per kg and day as a function of the WIMP mass. The horizontal lines are the exclusion limits.

be lower than  $7 \times 10^{-22} e \text{ cm}$  for WIMP mass of 100 GeV to satisfy the current experimental exclusion limits. Although we consider that the interaction of EDM is suppressed by CP violation, the suppression could be compensated by the enhancement of spin independent interaction. WIMP could thus be detected by the EDM interaction in near future if the suppression is not seriously small. The discovery of WIMP by EDM interaction can also be the key to disclose the nature of CP violation nature.

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